Investigating Incentive-Compatible AMMs in Binary Prediction Markets

Arya Maheshwari[1], Abiram Gangavaram[1], Janum Shah[1]

1. Department of Computer Science, Princeton University

"Blockchain-based prediction markets may be the one force strong enough to counterbalance the spread of incorrect information on social media."
- Balaji S. Srinivasan
Former CTO of Coinbase

Background

PREDICTION MARKETS

- Prediction markets: financial markets where participants can trade securities corresponding to outcomes of certain events.
- DeFi motivation: unlike traditional markets, where one centralized party controls the pricing, decentralized markets allow market participants to collectively price assets based on their beliefs.
- We focus on *binary prediction markets*, where exactly one of *n* initial possible events will be realized (one security pays out).

SCORING RULES:

- Scoring rules are used in prediction markets to quantify how to reward participants for the accuracy of their bets.
- To elicit the best information, scoring rules should be designed to reward participants for honest predictions. A scoring rule is *strictly proper* (or *incentive compatible*) if it is optimal for betters to report their values truthfully.

AUTOMATED MARKET MAKERS (AMMs)

- In general, any financial market may suffer from a lack of liquidity.
- Solution: to continuously price assets, some markets leverage Automated Market Makers (AMMs), which use specific algorithms to fairly update asset prices.
- In AMMs, buyers/sellers can always interact with an asset.

Theory

For a set X of outcomes, a scoring rule is a function $S(\mathbf{x}, i) \in \mathbb{R}$ where $i \in X$ is the outcome, x is a probability distribution over X.

- Strictly proper if for a bettor's true belief $\mathbf{p}$,

$$\mathbf{p} = \max_x \mathbf{E}_{i \sim \mathbf{p}}[S(\mathbf{x}, i)]$$

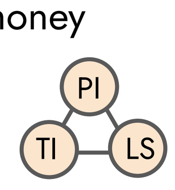
- One of the most widely-used strictly proper rules is the logarithmic market scoring rule (LMSR): $S(\mathbf{x}, i) = \log \mathbf{x}_i$

Framing in terms of AMMs:

- Scoring rules can be implemented in practice through AMMs.
- Market state: a *quantity vector* $\mathbf{q}$, with elements q_j representing shares for the *j*-th outcome that traders can buy or sell at prices $p_j(\mathbf{q})$.
- Key idea: for correctly set prices, the trader faces the same incentives with the AMM as the desired MSR (e.g. see [2], *T5.1*). A LMSR AMM has $\mathbf{p}(\mathbf{q}) = \nabla C(\mathbf{q})$ for $C(\mathbf{q}) = b\log(\Sigma_i e^{q_i/b})$.
- This AMM is expected to run at a loss if final market values are more accurate than the initial (viewed as the "price of information").

Desirable AMM properties:

1. **Path independence** (PI): transaction cost depends only on endpoint states, not path (mini-transactions) in between ⇒ no "money pump."
2. **Translation invariance** (TI): $\Sigma_i p_i(\mathbf{q}) = 1$. Then price can be interpreted as a *probability* of an outcome.
3. **Liquidity sensitivity** (LS): if price elasticity adjusts based on market volume/activity. Formally: LS if $\mathbf{p}(\mathbf{q} + k\mathbf{1}) \neq \mathbf{p}(\mathbf{q})$



Key result *(Othman et. al '13)*: impossible for an AMM to have all 3 properties!
Mathematical facts: any AMM with $\mathbf{p}(\mathbf{q}) = \nabla C(\mathbf{q})$ for *some* cost function C(q) is necessarily path independent. **LMSR AMM satisfies (1)+(2), *not (3)*.**

From LMSR to LS-LMSR
Othman et. al proposed relaxing (2) in favor of (3) to obtain the **LS-LMSR AMM**, with $C(\mathbf{q}) = b(\mathbf{q})\log(\Sigma_i e^{q_i/b(\mathbf{q})})$, for $b(\mathbf{q}) = \alpha \Sigma_i\, q_i$. (This is LS because b now is a function of $\mathbf{q}$, rather than constant).

According to their theoretical analysis, the LS-LMSR AMM offers benefits both as a more realistic model that responds to liquidity changes and also in terms of revenue: while LMSR generally runs at an expected loss, the LS-LMSR AMM loss can always be made arbitrarily small and can expect a profit under certain final states of the market.

Our project goal: compare this proposed LS-LMSR AMM with an LMSR AMM in practice, by simulating a binary prediction market. This analysis will provide a baseline for benefits and drawbacks of using such AMMs in practice, particularly in the setting of DeFi.

Experimental Approach

SIMULATING A BINARY PREDICTION MARKET
To investigate the characteristics of LS-LMSR AMM applied to a binary prediction market compared to those of an LMSR AMM, we conducted a simulation-based study in Python. Each simulation proceeds with the following steps:

Generate Probabilities	Simulate	Collect Metrics
Randomize and generate a "ground truth" probability that represents the true probability of some binary outcome. Draw the initial trader beliefs from a distribution around this ground truth with some level of noise.	Traders proceed sequentially, and each trader's belief is updated based on the current market perception (both LMSR and LS-LMSR). Traders play truthfully, buying shares of an outcome until the price exceeds their belief.	Draw an outcome and calculate the payouts and the AMM's revenue. Determine the accuracy of the market by interpreting the final market prices as probabilities and comparing them to the ground truth.

EXPERIMENTAL CONDITIONS

- We designed a "Noisy Information Market," modeling each trader's beliefs as a weighted average of (1) "private information" drawn uniformly from an interval around the ground truth and (2) an average of previous market beliefs. We update the weighting over time so traders are increasingly influenced by market perception as time goes on, representing a converging market belief.
- We varied α for the LS-LMSR AMM and number of traders to investigate the impact of these parameters on the different AMMs performance and accuracy.

Preliminary Results

Figure 1

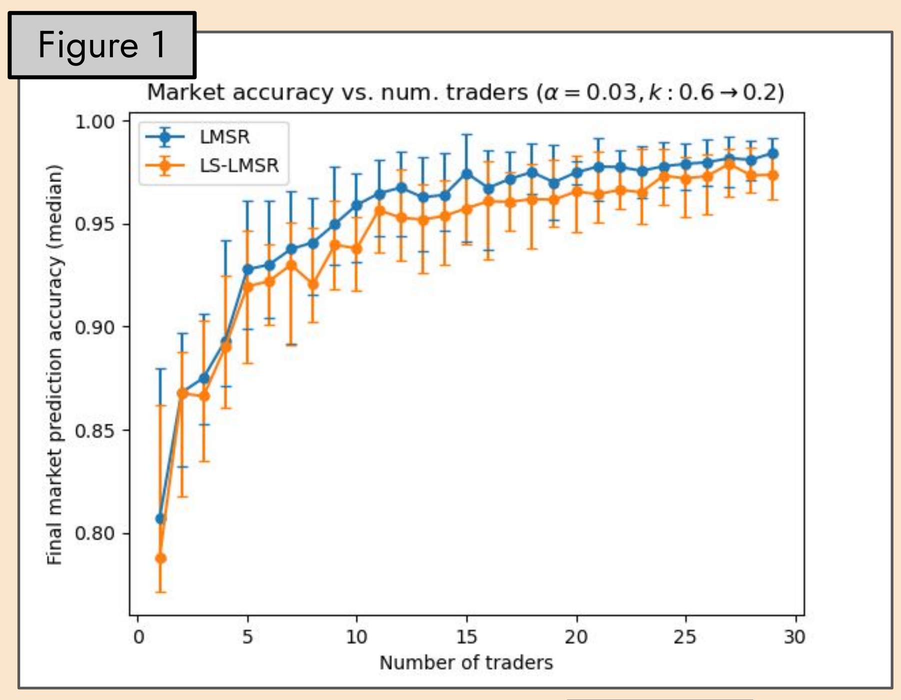


Fig. 1 demonstrates that final market accuracy improves when more traders participate (though with decreasing marginal improvement), aligning with our model design goal of trader prediction improving as more traders make bets and thus provide information via the market to future traders.

Fig. 2 illustrates the tradeoff that exists between price accuracy and revenue (accuracy decreases as revenue increases) and shows that LS-LMSR is more profitable in practice than LMSR but yields worse accuracies, as also seen in Fig. 1 for different numbers of traders.

Figure 2

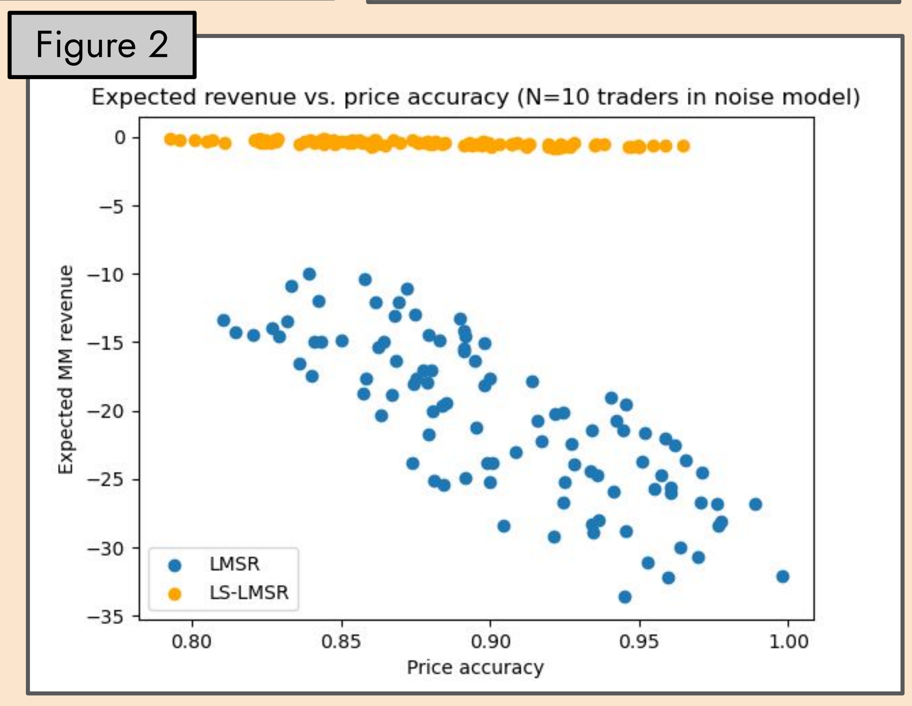


Figure 3

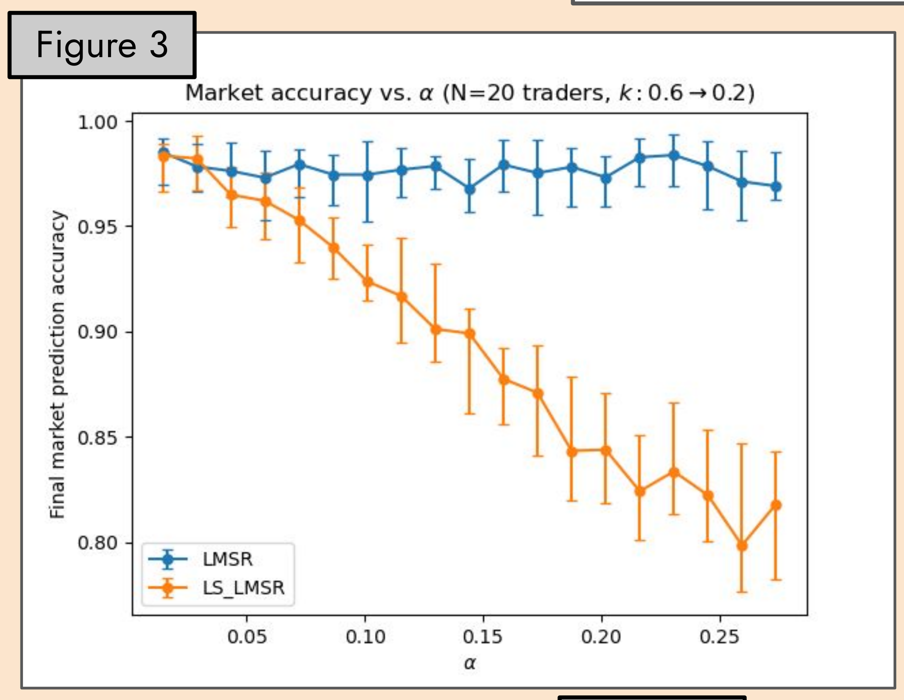
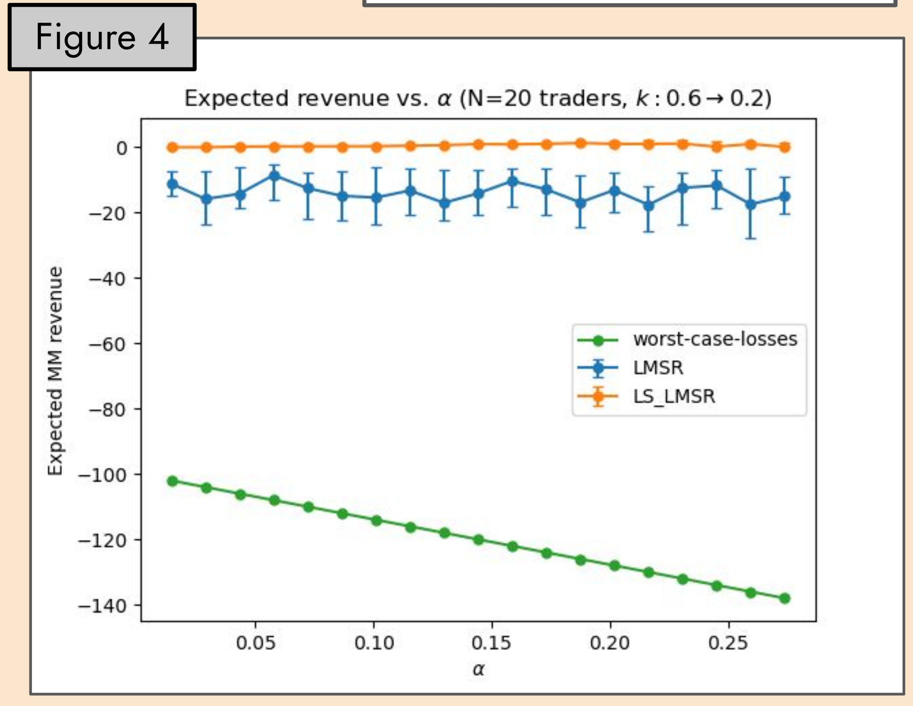


Fig. 3 shows accuracy for both AMMs across α, where models were compared by setting a corresponding b in LMSR to each α of LS-LMSR such that *worst-case loss* was equal. LMSR stays near perfect throughout while LS-LMSR accuracy drops significantly as α increases (possibly due to higher prices deterring traders from making reports).

Fig. 4 shows final expected revenue for LMSR and LS-LMSR across α. Interestingly, we see no significant trend across α: while Othman et al. suggest revenue for the LS-LMSR model should in theory increase with α, they note results in practice may be ambiguous, as we find here, due to competing influences.

Figure 4



Current DeFi Applications

AUGUR
Augur is a decentralized prediction market platform that is built on Ethereum. Users can create their own prediction markets by leveraging Augur's smart contracts. This not only allows anyone to participate in prediction markets, but also allows anyone to create a prediction market of their choosing. In markets created with Augur, the underlying AMM is powered by LS-LMSR.

GNOSIS
Gnosis is a prediction market platform also built on Ethereum. Gnosis offers smart contracts for two different AMMs; one of which is powered by LMSR. Similar to Augur, users can leverage the smart contracts based on these protocols to create their own prediction markets.

ZEITGEIST (RIKKIDO)
The Rikkido scoring rule aims to fix a flaw common to both LMSR and LS-LMSR: they both require some variable parameter that needs to be tuned by the market maker. Market makers who use Rikkido can leverage this ability to quickly and easily set up multiple markets.

Future Work

- Implement and compare the characteristics of Zeitgeist's Rikkido Scoring Rule with LMSR and LS-LMSR.
- Explore the on-chain implementation of these AMMs (LMSR, LS-LMSR, and Rikkido) via smart contracts.
- Explore α parameter variation for LS-LMSR and the liquidity parameter b for LMSR, which could provide insights into the sensitivity of AMMs' parameters to different parameter choices.
- Inject real-world data from binary prediction markets to complement our simulation-based approach and provide empirical evidence on AMM's performance in real-world environments.

Discussion

- As the number of traders participating in the market increases, the market accuracy improves — aligning with our design goal of information aggregation through trader participation.
- This marginal improvement decreases as the number of traders increases, suggesting that there is an optimal number of traders to achieve the best accuracy in prediction markets.
- There is a clear tradeoff between price accuracy and revenue, and LS-LMSR seems to be more profitable but less accurate than LMSR.
- This tradeoff suggests that revenue-maximizing strategies have implications for prediction market outcomes, and further investigation into the tradeoff between accuracy and revenue in AMMs is warranted.
- As the α parameter increases, LS-LMSR accuracy tends to decrease significantly, possibly due to a higher α (thus higher prices) deterring trader participation and thus reducing information in the market. This highlights the sensitivity of LS-LMSR's performance to the α parameter.

Acknowledgements & References

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